Probing The Non-Universality of The Halo Mass Function

Project Report
IAS - Summer Research Fellowship 2017
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Abstract

The excursion set theory used by Press and Schechter gives a fitting form for the mass function of dark matter haloes. This was improved upon by using an ellipsoidal collapse model by Sheth and Tormen, who provided universal (independent of cosmology and power spectrum) fitting functions. We aim to investigate the non-universality of the Sheth-Tormen fitting functions through a suite of N-body simulations using the GADGET-2 code, specifically the impact of the power spectrum on the mass function parameters. We study the mass function with a power law power spectrum in an Einstein-deSitter universe, which is a scale-free cosmology. Thus, the dependence of fit parameters on power spectrum can be isolated.

1 Theory

1.1 Spherical Collapse

The currently accepted mechanism for the formation of haloes is the "hierarchical" model, where due to pressureless collapse, haloes form at the smallest scales, and then merge to form bigger haloes.

To study the formation of dark matter haloes, we use a model called *Spherical Collapse*. We assume haloes to be almost spherical overdense dark matter clumps existing in a pressureless universe of critical density. We begin with a spherical perturbation of radius R and initial overdensity δ evolving in this background universe. We can treat the perturbation as a separate universe expanding in an Einstein-deSitter background. Such a universe collapses after some time and stays stable and bound in the absence of pressure.

Let us consider the growth of a sphere under its own gravity. The governing equation for the motion of its radius will be:

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} \tag{1}$$

Integrating over r once, it becomes:

$$\dot{r}^2 = \frac{2GM}{r} + C \tag{2}$$

This differential equation has the following parametric solution:

$$r = A(1 - \cos\theta) \tag{3}$$

$$t = B(\theta - \sin\theta) \tag{4}$$

$$A^3 = GMB^2 \tag{5}$$

Considering the behaviour of this solution at early times, when $\theta \to 0$, and expanding terms, we have $r = A\theta^2/2$ and $t = B\theta^3/6$. Eliminating θ, we have $8r^3/A^3 = 36t^2/B^2$, which gives us $r^3 = (9/2)GMt^2$. Since $r^3 = 3M/4\pi\rho$, we have $6\pi G\rho = t^{-2}$. Recall here, that in a single-component universe with $\Omega = 1$,

$\rho \propto t^{-2}$. Thus, at early times, our spherical perturbation evolves exactly as a single-component universe. Also, note here that we can write:

$$r = \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3} \tag{6}$$

Moving to a higher value of time, we can expand the expressions for r and t further, to get:

$$r = \frac{A\theta^2}{2}\left(1 - \frac{\theta^2}{12}\right) \tag{7}$$

$$t = \frac{B\theta^3}{6}\left(1 - \frac{\theta^2}{20}\right) \tag{8}$$

Solving for $r(t)$, we have,

$$r = \frac{A}{2}\left(\frac{6t}{B}\right)^{2/3}\left[1 \mp \frac{1}{20}\left(\frac{6t}{B}\right)^{2/3}\right] \tag{9}$$

We can see that the first term on the right hand side of (9) is the same as equation (6). Thus, the first term represents the first-order expansion, and the second term shows the growth of the density perturbation.

With this done, let us consider the mass of expanding sphere. The initial mass is $M = 4\pi\rho r^3/3$. Let us say the mass is disturbed by an overdensity of magnitude δ. To conserve mass in the system, the radius must change by the infinitesimal amount δr. By the conservation of mass, we have,

$$M = \frac{4\pi}{3}\rho r^3 = \frac{4\pi}{3}\rho r^3(1+\delta)(1+\delta r)^3 \tag{10}$$

Which becomes

$$(1+\delta)(1+\delta r)^3 = 1 \tag{11}$$

Expanding this and keeping only first-order terms, we get,

$$\delta \approx -3\delta r \tag{12}$$

Looking at (9), we can expand the right hand side a $r_0 + \delta r$, and then substitute it in (12) to get δ as a function of time:

$$\delta = \pm \frac{3}{20} \left(\frac{6t}{B} \right)^{2/3} \tag{13}$$

Let us now revisit the behavior of the parametric solutions. We can differentiate r with respect to θ to obtain the parametric points that correspond to extrema of r. These come out to be $\theta = 0, \pi, 2\pi$.

We know, that at $\theta = 0$, the sphere is undergoing Hubble expansion and $r = 0$. Post this, at $\theta = \pi$, there is a turnaround, where the radius is maximum. After this begins the collapse of the sphere, which terminates at the point $\theta = 2\pi$.

Using (13) at the parametric points $\theta = \pi$ for turnaround, and $\theta = 2\pi$ for collapse, we can calculate the linear theory predictions for the overdensities at turnaround and collapse. For $\theta = \pi$, we have $t_{\text{turn}} = \pi B$, and for $\theta = 2\pi$ we have $t_{\text{collapse}} = 2\pi B$.

$$\delta_{\text{turn}} = \frac{3}{20} \left(\frac{6t_{\text{turn}}}{B} \right)^{2/3} = \frac{3}{20} (6\pi)^{2/3} = 1.06 \tag{14}$$

$$\delta_{\text{collapse}} = \frac{3}{20} \left(\frac{6t_{\text{collapse}}}{B} \right)^{2/3} = \frac{3}{20} (12\pi)^{2/3} = 1.686 \tag{15}$$

Thus, we can see that when the overdensity predicated by the linear model approaches the order unity, the collapse of the overdensity begins, and it finally collapses at a critical value of overdensity which is 1.686.

1.2 The Halo Mass Function

Haloes of dark matter are the basic units of large-scale structure, and a successful tool for the verification of the theories of structure formation is the halo mass function, which denotes the spectrum of fully-formed, or virialized halos that form out of an initial field of overdensities.

The mass function gives the number density of virialized haloes in the mass range m and $m + dm$ at some redshift. It can be calculated observationally by selecting a volume in space and counting the number of objects of a given mass in that region, and it is now an important tool in various aspects of cosmology and astrophysics, including normalization of the power spectrum, the characteristics of the overdensity field, and in galaxy formation, apart from being a verification tool for theoretical models in cosmology.

1.2.1 Press-Schechter Formalism

Press and Schechter [1] were the first ones to provide a process to obtain the mass distribution of haloes from an underlying density field. In this section, we will review their formalism and results, along with certain extensions.

Press and Schechter (PS hereafter) assumed that haloes form at the peaks of the field of overdensities. They said that if the overdensity field is smoothed on a scale of the radius which corresponds to a given mass M, haloes form in the portions of space where the overdensity exceeds the critical overdensity of 1.686 (from (15)). They say that collapsing perturbations follow linear theory till this critical value, and suddenly collapse to form haloes. This claim lacks mathematical rigour but turns out to be a reasonable approximation that works well.

First of all, we look at an equivalent length scale r to a mass m, which is given by the relation:

$$m = \frac{4\pi}{3}\rho r^3 \tag{16}$$

Then, the variance of the density field corresponding to a mass m is the variance under a smoothing function of radius r, which is σ_r, which we shall alternatively denote by $\sigma(m)$.

To quantify the smoothing, we multiply the power spectrum by a smoothing function with a characteristic scale R, denoted by $W(kr)$ and then integrate:

$$\sigma_r^2 = \int \Delta^2(k) W^2(kr) d\ln k \tag{17}$$

Which we can also write as:

$$\sigma_r^2 = \int \frac{k^3}{2\pi^2} P(k) W^2(kr) \frac{dk}{k} \tag{18}$$

The probability distribution of finding a density contrast filtered over this scale is Gaussian, as the underlying field, and takes the form:

$$p(\delta; m) = \frac{1}{\sqrt{2\pi\sigma^2(m)}} \exp\left(- \frac{\delta^2}{2\sigma^2(m)} \right) \tag{19}$$

To find the fraction of collapsed mass, as per the theory of PS, we need to integrate this distribution from the critical value of δ_{cr}:

$$P(> \delta_{cr}) = \int_{\delta_{cr}}^{\infty} P(\delta; m) d\delta = \int_{\delta_{cr}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2(m)}} \exp\left(- \frac{\delta^2}{2\sigma^2(m)} \right) d\delta \tag{20}$$

Which means,

$$F(> m) = \frac{1}{2}\left[1 - \mathrm{erf}\left(- \frac{\delta}{\sqrt{2}\sigma(m)} \right) \right] \tag{21}$$

Noting that this is normalised to 1/2, PS realised this meant only half of the universe was available for collapse, which was due to the Gaussianity of the distribution. To resolve this, they multiplied this fraction by an ad-hoc factor of 2.

The fraction of collapsed objects in the mass range from m to $m + dm$ is then given by simply subtracting the fractions at the two points. So,

$$f(m)dm = F(> m + dm) - F(> m) \tag{22}$$

Therefore,

$$f(M) = \left| \frac{F(> m + dm) - F(> m)}{dm} \right| = \left| \frac{dF(m)}{dm} \right| \tag{23}$$

To get the number density of objects per unit mass interval, we multiply by the density and divide by the mass of one object, and we get:

$$\frac{dn(m)}{dm} = \frac{\rho}{m} \left| \frac{dF(m)}{dm} \right| \tag{24}$$

To obtain the functional form, we now decompose the mass derivative operator as:

$$\frac{d}{dm} = \frac{d\sigma}{dm}\frac{d}{d\sigma} \tag{25}$$

We can then write the number of halo objects as:

$$\frac{dn(m)}{dm} = \frac{\rho}{m}\left|\frac{d\sigma(m)}{dm}\right|\sqrt{\frac{2}{\pi}}\frac{\delta_{cr}}{\sigma^2(m)}\exp\left(-\frac{\delta_{cr}^2}{2\sigma^2(m)}\right) \tag{26}$$

And the number of haloes with mass greater than m_0 as:

$$N(>m_0) = \int_{m_0}^{\infty}\frac{dn(m)}{dm}dm \tag{27}$$

In conclusion, we can write a general form for the number of haloes, as:

$$\frac{dn(m)}{dm} = \frac{\rho}{m}\left|\frac{d\ln\sigma(m)}{dm}\right| f(\nu) \tag{28}$$

Where $f(\nu)$ is formally labelled the mass function, and $\nu = \delta_c/\sigma(m,z)$. It is convenient to specify different models in terms of different fitting forms / expressions for the mass function.

Comparing directly with (26), we can see that the Press-Schechter model supplies the mass function

$$f(\nu) = \sqrt{\frac{2}{\pi}}\nu\exp(-\nu^2/2) \tag{29}$$

1.2.2 Sheth-Tormen Formalism

Using the formalism of Sheth and Tormen [2], we can obtain a fitting form for the mass function that better matches large-scale simulations.

In their paper, Sheth, Mo and Tormen (SMT hereafter), assert that the collapse of a halo depends not only on its initial overdensity, but the surrounding shear field also. Rather than assuming spherical collapse, they work with a tri-axial ellipsoidal collapse model, where it is assumed that the final collapse happens when the third axis collapses.

SMT characterise the collapse in terms of initial overdensity δ, ellipticity e and prolateness p. They express the barrier shape, which is constant in the PS formalism, as a function of time, and finally give the fitting form:

$$f_{ST}(\nu) = A\sqrt{\frac{2q}{\pi}}\left[1 + \left(q\nu^2\right)^{-p}\right]\nu \exp\left[-\frac{q\nu^2}{2}\right] \tag{30}$$

We employ the condition that all mass should be in haloes, and thus the mass function can be normalised:

$$\int_0^\infty \frac{1}{\nu} f(\nu) d\nu = 1 \tag{31}$$

The parameter A is thus given in terms of the parameter p as:

$$A = \left[1 + \frac{2^{-p}\Gamma(0.5 - p)}{\sqrt{\pi}}\right]^{-1} \tag{32}$$

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2 Cosmological N-Body Simulations

2.1 Introduction

We know from theories of structure formation that small perturbations in mass density amplified by gravity lead to the formation of structure in the universe. In the linear regime, and in situations with a high amount of symmetry, we can solve analytic equations to study the growth of these. However, post-the quasi-linear regime, and in a more general set of situations, it becomes increasingly difficult to study structure formation analytically.

Here, the use of N-Body simulations is instrumental to modern cosmology. They are used to study the evolution of perturbations in highly non-linear regimes, and form an indispensable tool for testing theory and comparing with observations.

In writing N-Body codes, one has to keep in mind some physical requirements [3]:

- The simulation volume cannot be assumed to exist in isolation, and the outside of it has to be accounted for. For this, periodic boundary conditions are used. In this case the most natural geometry for the simulation volume is the cube.
- The evolution of perturbations should be independent of the boundary conditions.
- The average density over the box should be equal to the average density of the universe.
- Perturbations averaged over the scale of the box must be of the order of zero.
- The interactions of a large group of particles are approximated by considering a single particle. Thus their interactions must be collisionless.
- The mass of a single 'particle' must be smaller than the smallest structures we want to investigate. Thus, the number of particles is very high for achieving a mass resolution that enables us to probe scales relevant to non-linear evolution.

N-Body codes evolve the simulation volume after taking in a set of initial conditions consisting of the perturbation and velocity fields at a given starting time. Subsequently, they work through two steps. Considering a Newtonian N-Body problem, we have two sets of equations. First, we have the computation of force on each particle due to all other particles, and secondly, we have the equations of motion for each particle under the forces on it. A cosmological N-Body code works in the same modules. One module performs a force computation for each particle, and the other module then updates the positions and velocities of the particles.

2.2 Equations of Motion

In an expanding universe with scale factor a, we can write equations for a collections of particles that interact only through gravity as [3]:

$$\ddot{x} + 2\frac{\dot{a}}{a}\dot{x} = -\frac{1}{a^2}\nabla_x \phi \tag{33}$$

$$\nabla_x^2 \phi = 4\pi G a^2 \bar{\rho}\delta = \frac{3}{2} H^2 \Omega_m^2 \frac{\delta}{a} \tag{34}$$

N-Body codes seek to integrate these equations numerically, and then update the positions and velocities of each particle at every step. The computational complexity of this step is $\mathcal{O}(n)$. The accelerations and velocities are expressed as discrete-time derivatives of positions, and Euler's method of solving differential equations is used along with the Leap-Frog integration method. The error is of the order of the square of the time step, which is chosen so as to keep momentum conserved.

2.3 Particle Distribution

We look at a system with a high density of dark matter particles, and we characterise their distribution by the single-particle distribution function, which is the mass density of particles in phase space $f(\mathbf{x}, \mathbf{v}, t)$. The collisionless Boltzmann equation describes the time evolution of this function [5].

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial \mathbf{x}}\frac{\partial f}{\partial \mathbf{v}} \tag{35}$$

Where ϕ is the gravitational potential. The Poisson equation then becomes

$$\nabla^2 \phi = 4\pi G \int f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \tag{36}$$

For a large number of particles, solving these coupled equations is a computational challenge. In an N-Body simulation, some N particles are sampled from the underlying distribution, and their discrete equations becomes

$$\ddot{\mathbf{x}}_i = -\nabla \phi \tag{37}$$

$$\phi(\mathbf{x}) = -G \sum_{1}^{N} \frac{m_j}{\sqrt{(x - x_j)^2 + \epsilon^2}} \tag{38}$$

The parameter ϵ is called the **softening length**, which ensures that the potential stays bounded and becomes constant at very short inter-particle distance, thus ensuring that the dynamics are collisionless. Thus, it provides the force resolution for the simulation.

2.4 Assigning Densities

We divide the simulation cube into cubical cells with side d. Each cell is assigned a mass depending on the mass of the particles that overlap with it. For a cell with centre $\mathbf{x}_c$, the mass contribution due to a particle at $\mathbf{x}_i$ is given by

$$W(\mathbf{x}_c - \mathbf{x}_i) = \int \Pi\left(\frac{\mathbf{x}' - \mathbf{x}_c}{d}\right) S(\mathbf{x}' - \mathbf{x}_i) d\mathbf{x}' \tag{39}$$

Taking $\mathbf{x}'$ as $\mathbf{x}$, this assigned weight becomes a convolution product of the window $W(x)$ and the shape function $S(x)$.

The total density at the grid point is the contribution summed over all particles.

$$\rho(\mathbf{x}_c) = \frac{1}{d^3} \sum_{1}^{N} m_i W(\mathbf{x}_c - \mathbf{x}_i) \tag{40}$$

GADGET-2 uses the **Cloud-In-Cell** interpolation scheme to assign the density field, the shape function for which is:

$$S(x) = \frac{1}{d^3} \Pi\left(\frac{x}{d}\right) \star \delta(x) A \tag{41}$$

2.5 Force Computation

We solve the Poisson equation and the equations of motion at every step. The calculation of forces on each particle is a time-consuming step in the simulations, and thus attention is paid to algorithms for reducing computational cost. The computation of force involves three steps [3]:

1. **Density Contrast:** Masses are assigned to mesh points by using an isotropic smoothing function as detailed above, and then the density contrast is calculatedby calculating the deviation from the average mass density at each point. Post this, an FFT is done to convert this density contrast field to the Fourier domain.

2. **Poisson Equation:** Next, the code solves the Poisson equation with periodic boundary conditions. This is done through a variety of methods, as enumer-

ated below.

3. **Gravitational Force:** The final step involves computing the gradient of the obtained potential to get the gravitational force. This can be done in the Fourier domain directly, or can be done in a direct way to compute a discrete derivative of the potential obtained at each mesh point.

Direct particle-by-particle force calculation includes $N(N-1)/2$ calculations for N particles, and becomes infeasible for a large number of particles, with a complexity of $\mathcal{O}(n^2)$. To bring down the computational cost to a logarithmic complexity, there exist various algorithms.

GADGET-2 uses a hybrid TreePM algorithm, that employs a Tree algorithm to compute short-range force, and the Particle-Mesh Method for long-range forces.

2.5.1 Particle-Mesh Algorithm

This method works by solving the Poisson equation in Fourier space, where the equation becomes algebraic. The computed density contrast is taken to the Fourier space using an FFT, and then the Green's Function method is used to find the potential. In real space, the potential is the convolution of the density and the Green's Function:

$$\phi(\mathbf{x}) = \int G(\mathbf{x} - \mathbf{x}')\rho(\mathbf{x})d\mathbf{x}' \tag{42}$$

In Fourier space this corresponds to a product

$$\tilde{\phi}(k) = \tilde{G}(k)\tilde{\rho(k)} \tag{43}$$

The Green's function in Fourier space is $\tilde{G(k)} = -4\pi G/k^2$.

2.5.2 Tree Method

This organises the particles in a hierarchical tree structure. The force exerted on a given particle by those particles is approximated by the lower order multipole moments of that group. The accuracy in the force is decided by the expressions in the multipole moment.

GADGET-2 uses up to a quadrupole term for the force. The simulation volume is divided into cubes, which are further divided into eight children. Each child holds one particle, and sends the subsequent ones allotted to it into further descendants. Each cube level holds data about the gravitational force contribution from the particles it contains. The tree is traversed and the forces added up. This is the Barnes-Hut tree method.

2.5.3 Gradient

The force is computed by taking the finite difference gradient of the potential function. Then, the force is de-interpolated back to the mesh points.

2.6 Integrating The Equations of Motion

The integration method used in cosmological simulations needs to be a set of symplectic transformations, to conserve the canonical relations, and thus the total energy of the system.

GADGET-2 uses the leap-frog method, which decomposes the evolution of the system into a series of "kick" and "drift" actions.

The Hamiltonian can be broken into kinetic and potential terms.

The drift operator pushes the position using velocities while keeping the momentum constant:

$$D(\Delta t) = \begin{cases} x_i \rightarrow x_i + \frac{p_i}{a^2 m_i}\Delta t \\ p_i \rightarrow p_i \end{cases} \tag{44}$$

The kick operator kicks the velocity (momentum) to an updated value using the force, keeping the position constant:

$$K(\Delta t) = \begin{cases} x_i \rightarrow x_i \\ p_i \rightarrow p_i + \sum_k \frac{m_i m_k}{a} F_{ik}\Delta t \end{cases} \tag{45}$$

Both the above transformations are keeping in mind comoving cooordinates. It can be easily checked that both these operations are symplectic.

GADGET-2 uses a "drift-kick-drift" (DKD) sequence to advance timesteps in the simulation:

$$U(\Delta t) = D\Big(\frac{\Delta t}{2}\Big)K(\Delta t)D\Big(\frac{\Delta t}{2}\Big) \tag{46}$$

2.7 Initial Conditions

N-Body simulations are usually begun from homogeneous initial conditions, with initial perturbations easily in the linear regime. Setting up these conditions requires the computation of velocity field and a perturbation field for the simulation contents.

The density field is related to the gravitational potential by the second equation in 34. The velocity field can also be related to the potential in the Zel'dovich Approximation. Using these two relations, once the potential is generated, initial density contrasts and velocities can be generated [4].

Since the density field is a Gaussian random field, the gravitational potential used for it is also statistically Gaussian. Such perturbations evolve independently in time and are completely characterised by their power spectrum. Thus, often, the power spectrum is used to generate initial density fields.

N-GenIC is a parallel code written by Volker Springel in 2003 for the generation of initial conditions for cosmological simulations with GADGET. It generates files in both GADGET-2 formats to be used as inputs.

The code is available from `http://www.mpa-garching.mpg.de/gadget/`.

This code utilises the Zel'dovich Approximation, and generates a power spectrum at the given initial redshift to construct the initial density and velocity fields. It can generate the power spectrum using three methods: 1) The Hu-Eisenstein Transfer Function, 2) The Efstathiou Parametrization, and 3) An input power spectrum generated from an outside source.

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3 Non-Universality of the Mass Function

The mass function $f(\nu)$ provided by the prescription of Press and Schechter [1] matches observations well for high masses, but deviates in the low mass range.

An improved functional form was given by Sheth, Mo and Tormen, by assuming ellipsoidal collapse instead of the standard spherical collapse model used by Press and Schechter.

The Sheth-Mo-Tormen model assumes that the collapse occurs independently along three axes, and furnishes a better fit function:

$$f_{ST}(\nu) = A\sqrt{\frac{2q}{\pi}}\left[1 + \left(q\nu^2\right)^{-p}\right]\nu \exp\left[-\frac{q\nu^2}{2}\right] \tag{47}$$

This function is supposedly universal: independent of the underlying statistics of the dark matter density field, and also independent of the background cosmology. However, recent simulations have indicated that the parameters in this functional form are in fact not universal [6] . The background cosmology influences the mass function by modifying the collapse threshold [7] .

Standard ΛCDM models have a primordial index that is a function of scale. The presence of the cosmological constant also changes the threshold overdensity for collapse. These complex effects make it hard to investigate the impact of the power spectrum on the mass function.

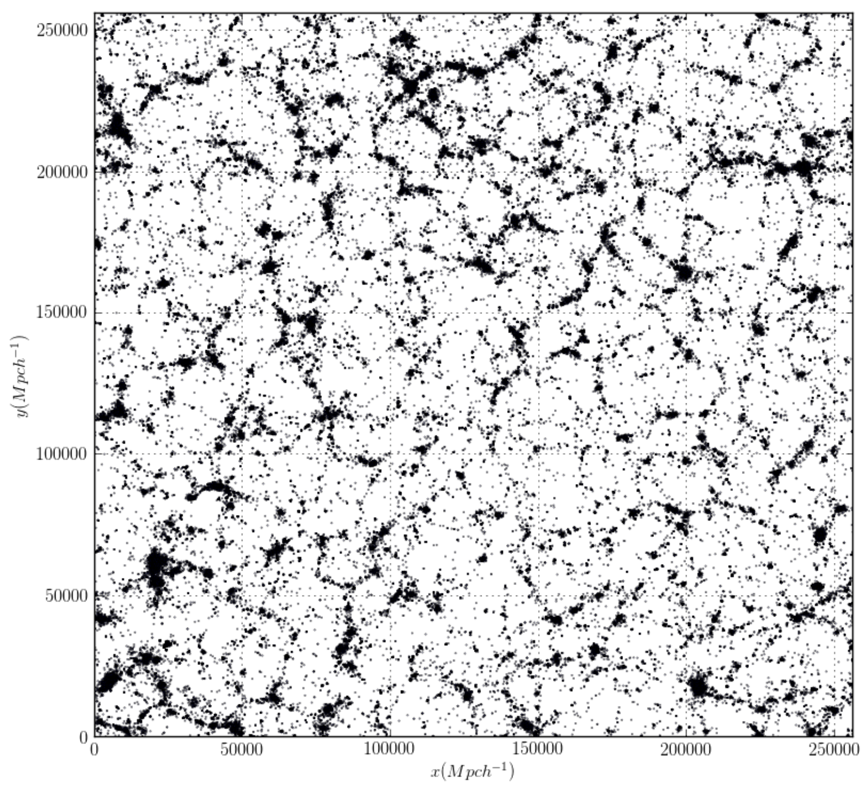
3.1 Method and Simulations

The power spectra for standard cosmologies introduces various characteristic scales, which make the analysis of the impact of the underlying statistics of dark matter on the mass function difficult.

We thus use a cosmology with power law power spectra $P(k) = Ak^n$, which is scale invariant, and thus a good tool to check whether the non-universality of the mass function can be connected to the power spectral index. The background cosmology is chosen to be Einstein-deSitter, where $\Omega_m = 1$. Due to this, the overdensity threshold is also independent of time.

We ran a suite of cosmological N-body simulations using the publicly available massively parallel code GADGET-2. The initial conditions were created through the code N-GENIC, which uses a glass file to sample from an underlying Gaussian distribution once a power spectrum is supplied to it.We used the power law power spectrum with a cutoff including for suppressing power at small scales:

Figure 1: A slice of the simulation box showing the formation of halos



$$P(k) = k^n \exp(-k^2 l^2)$$

Where $l = L/10$, where L is the particle-mesh grid length of the simulation.

We normalise the RMS mass fluctuations in a spherical tophat window of radius 8 Kpc h^{-1} to unity at current time ($a = 1$).

$$\sigma_8 = 1 \tag{48}$$

In an Einstein deSitter cosmology, the positive growing mode varies with scale factor as $D(a) = a$.

For a power law cosmology, the value of $\sigma(m,a)$ varies as:

$$\sigma(m,a) = \left(\frac{m}{m_{\rm nl}(a)}\right)^{-\frac{n+3}{6}}$$

Where $m_{\rm nl} = \frac{4}{3}\pi\rho r_{\rm nl}^3$.

The non-linear scale evolves with time as $r_{\rm nl} \propto D^{2/(n+3)} = a^{2/(n+3)}$.

Hence, we can segregate the time dependence of $\sigma(m,a)$ as $\sigma(m,a) = a\sigma(m)$. Where

$$\sigma(m) = \left(\frac{m}{m_{\rm nl}}\right)^{-\frac{n+3}{6}} \tag{49}$$

We start our simulations from an initial redshift where the RMS mass density fluctuations in a sphere of 1 Kpc h^{-1} were equal to 0.05 times the σ_8 value.

$$0.05 = \left(\frac{1}{8}\right)^{-(n+3)/2} a \tag{50}$$

Which means

$$1 + z_i = 20 \times 8^{(n+3)/2} \tag{51}$$

The mean interparticle distance is taken to be one unit distance, which in our case is equal to 1 Mpc h^{-1}. The softening length is taken to be $\epsilon = 0.03$ of the grid length, which in our case becomes 30 Kpc h^{-1}. The particle-mesh grid used for short-range force computation has a grid side of 1 Mpc h^{-1}.

The following table shows the suite of simulations.

n	L_{box}	N_{part}^3	z_i
$+0.0$	256	256^3	451.58
-0.5	256	256^3	268.08
-1.0	512	512^3	159.00
-1.2	512	512^3	128.96
-1.5	512	512^3	94.13
-1.8	1024	1024^3	68.64
-2.0	1024	1024^3	55.56

3.2 Analysis and Results

The counts of halos per logarithmic bin of mass $dn/d\ln m$, is given by

$$\frac{dn}{d\ln m} = \frac{\rho}{m}\frac{d\ln\sigma^{-1}}{d\ln m}f(\nu) \tag{52}$$

For our power law cosmology, we can thus write, using (49)

$$f(\nu) = \frac{6}{n+3}\frac{m}{\rho}\frac{dn}{d\ln m} \tag{53}$$

Where $\nu = \delta_{\rm cr}/D(a)\sigma(m) = \delta_{\rm cr}/a\sigma(m)$.

We use the method of the Friends-of-Friends (FOF) algorithm [8] to identify halos from our simulation box. We use a linking length $b = 0.2$ in grid units, and restrict our analysis to halos with a minimum of 60 dark matter particles. Figure 2 shows the evolution of the most massive halo in the simulation for the case $n = 0$.

We construct mass bins of logarithmic interval $\Delta\log m = 0.2$, and populate them with halos to construct an estimate of $dn/d\ln m$. We assume Poisson errors for halo counts in a bin. We then use this to plot $f(\nu)$.

We fit the obtained function f to the form of the Sheth-Tormen mass function. This functional form has two free parameters, p and q. Standard values are given in literature by $p = 0.3$ and $q = 0.707$ [9].

We use the method of χ^2-minimisation to best fit our data points to the form of the mass function and obtain parameters p and q.

For the case $n = 0$, the best fit parameter value are given by the following table,

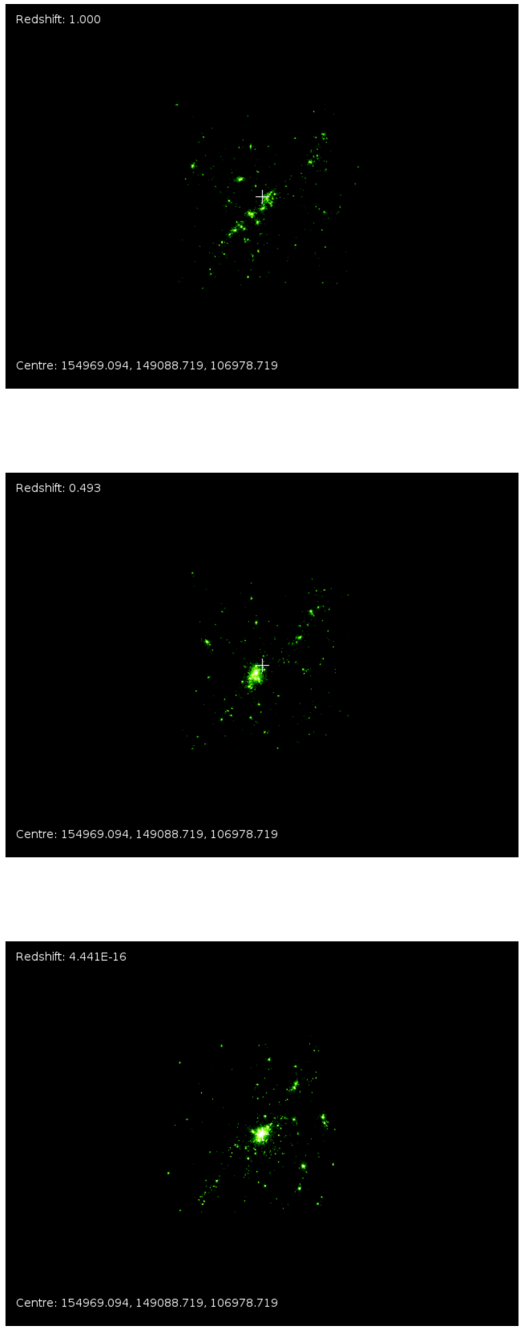


Figure 2: A view of the most massive halo in the simulation box, charting its evolution through time. Brightness indicates dark matter density.

along with their minimum χ^2 value.

z	p	q	$\chi^2_{\min}$
0.0	0.221	1.117	2.287
0.5	0.208	1.107	1.313
1.0	0.194	1.121	3.181
3.0	0.156	1.090	8.144

Figure 3 shows the confidence contours and fits for the mass function for snapshot outputs at four different redshifts.

We see that the values of the parameters p and q deviate significantly from the standard. The slight run of p with redshift is attributed to the imperfect index of the realisation of the power spectrum. Nevertheless, within the 3-σ significance level, the parameters form a good fit.

Future work includes completing the simulations and analysis for a range of values of n, and then trying to establish a fit between the values of p and q and n, in order to formalise the non-universality of the halo mass function.

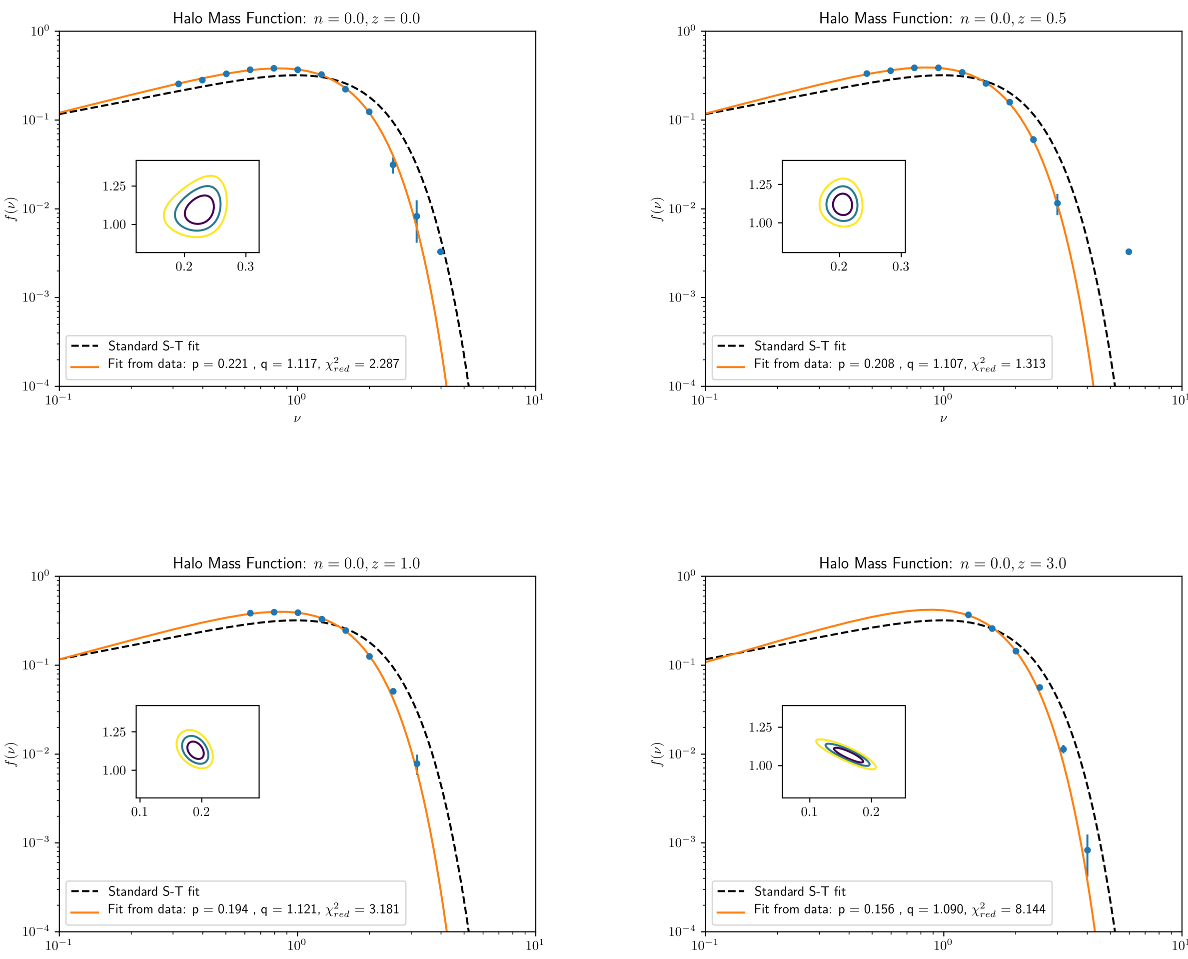


Figure 3: Halo Mass Function fits for halos generated from four snapshots, with 1,2,3 - σ confidence contours. Sheth-Tormen standard fit function shown for comparison.

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